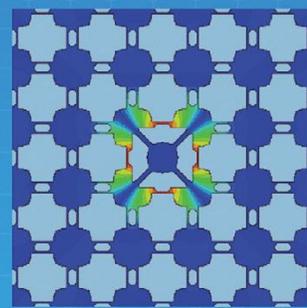
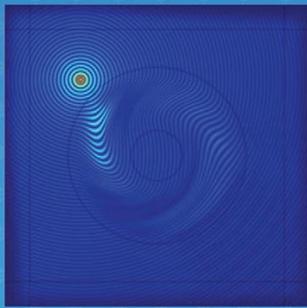
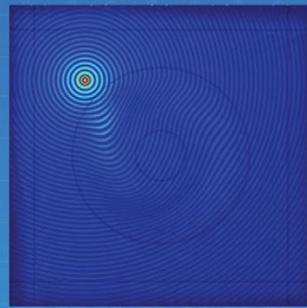
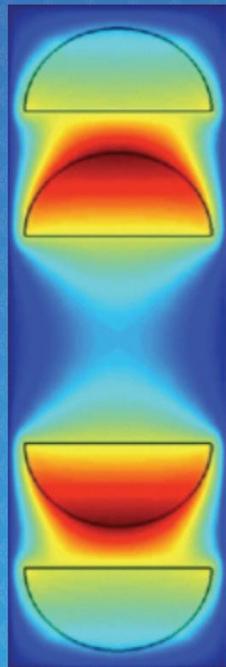
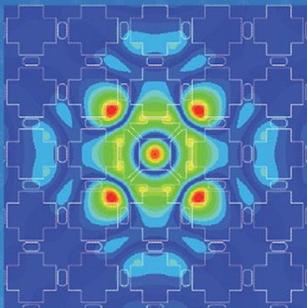


**SPIE.**

# THEORY AND DESIGN OF METAMATERIALS

**P. Frank Pai**  
**Guoliang Huang**  
*Editors*



# Theory and Design of Acoustic Metamaterials

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# Preface

Phononic crystals and metamaterials are artificial materials that are engineered to have properties that can manipulate and control the dispersive properties of waves and vibration. The transcendental properties of metamaterials are from their engineered constructs using artificially fabricated, extrinsic, low-dimensional inhomogeneities. This concept motivates engineers to dream and think beyond the constraints imposed by the performance limitations of conventional materials.

Elastic metamaterials (EMMs), or acoustic metamaterials, are designed by building periodically distributed subunits to create the desired, unnatural material properties to manipulate the dispersive properties of elastic waves, and they are often band gap structures. Due to the presence of the resonators, EMMs feature subwavelength wave manipulation capabilities, i.e., they modulate the propagation of waves with wavelengths significantly larger than the characteristic size of their unit cells. This opens opportunities for low-frequency wave control without the need to scale the structures to unmanageable sizes. Unfortunately, modeling and analysis of acoustic/elastic metamaterials is challenging because they are complicated built-up composite structures. In the literature, only a few simple models for acoustic metamaterials have been proposed, and they are often based on spatial averaging of heterogeneous material properties over each subunit; hence, they are valid only for waves of wavelengths that are much longer than the size of the subunits. When a metamaterial is described by its averaged material properties over each subunit, it can be treated as a homogeneous material, and it may possess negative effective mass and/or stiffness. The effective dynamic material properties induce a metamaterial, consisting of nondispersive materials, to behave as a dispersive material and enable a useful yet mysterious subwavelength stopband that allows no waves within that frequency range to propagate forward. Most current designs of acoustic/elastic metamaterials are based on the resonant motion effect. To manufacture such metamaterials with tiny subunits with the goal of preparing stopbands, expensive manufacturing techniques are required, including micro- and nanomanufacturing techniques that are still under development.

Key challenges in the development of metamaterials and their applications are the design of new metamaterials with different subsystems to have many and/or wide stopbands, physics-based modeling and understanding of their working mechanisms, and experimental verification of such mechanics-based, elastic metamaterials. This book presents the most recent theoretical developments and numerical/experimental validations of new metamaterials and phononic crystals, for broadband absorption of elastic waves and vibrations in structures. The specific objectives are to present detailed modeling and analysis methods, to demonstrate absorption and/or guidance of elastic/acoustic waves in metamaterials structures, and to reveal the actual working mechanisms of different metamaterials.

The nine chapters constituting this book present an up-to-date survey and the results of many aspects of phononic crystals and acoustic/elastic metamaterials, including sound attenuation/absorption, extraordinary transmission, wave broadband mitigation, wave steering, cloaking via the transformation method, optimization of the phononic crystals, and active acoustic metamaterials. We hope the variety of subjects discussed in this book, and the way to handle them (theoretically, numerically, and experimentally), illustrates the richness of the emerging topic of acoustic/elastic metamaterials and will continue to initiate even more research activity and applications in the near future.

In Chapter 1, Su and Sun propose three types of broadband single negativity elastic metamaterials. They optimized these models in parametric studies and made comparisons to determine the elastic metamaterial with the broadest band gap. They found that frame bending/stretching is more appropriate than beam bending serving as a metamaterial resonator. In addition, by designing several resonators within the metamaterial unit cell, band gaps can be made even broader. They also validated the proposed metamaterial models by numerical simulations and found that by taking the unit cell to analyze the steady-state response, they can accurately determine the band gap region. Metamaterial designs were fabricated using an Eden 350 3D printer. A design guide for three single negativity metamaterials is also proposed.

In Chapter 2, Akl and Baz present the basic theories and the underlying phenomena governing the behavior of active acoustic metamaterial (AAMM) platforms that have tunable and programmable mechanical and acoustic properties. These AAMMs are intended to enable operation over wide frequency bands, adapt to a varying external environment, and more importantly morph from one functional configuration to another based on the mission requirements of the AAMM. With such unique capabilities, acoustic cloaks, beam shifters, perfect absorbers, and/or perfect reflectors can be physically realizable by simply programming the same AAMM platform to acquire the appropriate mechanical and acoustic properties that are necessary to achieve the desired functionality without the need for any changes in the physical hardware of the platform itself.

In Chapter 3, Chen, Zhu, Huy, and Huang first develop a new membrane model to investigate the dynamic behavior of a membrane-type metamaterial (MAM) with one and multiple attached mass resonators. The proposed model can provide highly precise analytical solutions of coupled vibroacoustic problems of the MAM, in which the finite mass effects on the deformation of the membrane can be properly captured. The MAMs can be designed to possess nearly total reflection for targeting low-frequency acoustic sources. Then, a theoretical vibroacoustic plate model is developed to reveal the sound energy absorption mechanism within the MAM, which cannot be properly investigated by using the classic membrane theory. Through the plate model, the MAM has been demonstrated to be a superabsorber for low-frequency sound. Finally, a novel concept is proposed for designing acoustic liners by integrating the MAMs with conventional honeycomb cores, especially for efficiently mitigating broadband engine noises (50 to 4 kHz). The microstructural effects are systematically investigated for design optimization.

In Chapter 5, Xiao and Wen explore the sound transmission loss (STL) behavior of metamaterial thin plates with locally resonant spring-mass resonators. Two analytical methods are developed for the calculation of normal/oblique incidence STL and diffuse-field STL. One is known as the plane wave expansion method; the other one is an effective medium method. Predictions by the two methods show very good agreement in the subwavelength frequency region. It is demonstrated that a metamaterial plate can achieve a much higher STL than a bare plate (with the same surface mass density) at frequencies within the mass-law region and the coincidence region. It is also shown that the frequency band with high STL in a metamaterial plate can be broadened remarkably by replacing a single resonator with multiple damped smaller-mass resonators.

In Chapter 6, Pai presents theories and guidelines for design of one- and two-dimensional metamaterial structures based on multifrequency vibration absorbers for absorption of broadband transverse waves. Each of the proposed metamaterial structures consists of a uniform isotropic beam/plate and small multimass subsystems at distributed locations over the structure to act as multifrequency vibration absorbers. For an infinite metamaterial structure, governing equations of a unit cell are derived using the extended Hamilton principle. For a finite metamaterial beam/plate, because these two methods cannot account for the influences of the boundary conditions and transient vibrations, full-size finite-element modeling and analysis are performed. The concepts of negative effective mass and stiffness and how absorber subsystems create two stopbands are explained in detail. Numerical simulations reveal that the actual working mechanism of the proposed metamaterial structures is based on the concept of conventional mechanical vibration absorbers. A set of guidelines for the design of wide- and multistopband metamaterials is provided at the end.

In Chapter 7, Assouar gives a general overview on phononic crystals (PnCs) and acoustic metamaterials based on the finite plate, and some of their potential and appealing applications. In the first part, a brief presentation of numerical approaches mainly used to compute the physical properties of phononic crystals and acoustic metamaterial will be described. Phononic crystals are also introduced and related to some relevant works on micro/nanomechanical devices based on PnC structures, which have a great potential to improve the performances of available devices used in wireless communications and sensing. In Section 7.3, acoustic metamaterials that are primarily based on a two-dimensional finite system (i.e., 2.5D) are presented.

In Chapter 8, Wang and Dong demonstrate the possibility to use topology optimization for design of two-dimensional PnCs and phoxonic crystals (PxCs), primarily concerning band gap engineering and the corresponding PxC devices. An overview of work on the topology optimization of photonic crystals (PtCs), PnCs, and PxCs is presented. Both single- and multiobjective optimization problems based on genetic algorithms (GAs), a nature-inspired optimization technique, are described in detail. They illustrate a class of topology optimizations for PnCs with two systems: the solid–solid and vacuum–solid material phases. They also extend topology optimization to PxCs and make clear the great adaptability of multiobjective optimization methods for designing PxCs. Furthermore, besides bandgap maximization, they present the designs of PxC devices for acousto-optical applications. Finally, they present the focus, directions, and challenges in this research field.

In Chapter 9, Chang, Hu, and Hu discuss the three existing formulations in a systemic manner. The first formulation is based on the material interpretation of coordinate transformation, where the spatial distortion is expressed by an arbitrary curvilinear coordinate system. The second formulation is based on mapping of the governing equation, where a spatial distortion is considered as a point-to-point mapping between two spaces, and all concerned equations are written in a global Cartesian system. The third formulation is based on deformation of physical quantity during spatial distortion. Based on the physical interpretation of form-invariance of the governing equation, it is assumed that the governing equation should retain the same form before and after spatial distortion in a local Cartesian system; the local affine transformation on the physical quantity is decomposed to rotation and pure stretch operations, and by applying the local form-invariance of the governing equation and energy conservation, the transformation material can be obtained. The transformation materials of electromagnetic, acoustic, and elastic waves are presented using different formulations; some important applications with transformation materials are also provided. Finally, simplification and optimization of transformation

materials through adjusting spatial mapping have been examined for ease of material fabrication.

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*Frank Pai  
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October 2015*

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